Chaotic transport in the symmetry crossover regime with a spin-orbit interaction

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We study a chaotic quantum transport in the presence of a weak spin-orbit interaction. Our theory covers the whole symmetry crossover regime between time-reversal invariant systems with and without a spin-orbit interaction. This situation is experimentally realizable when the spin-orbit interaction is controlled in a conductor by applying an electric field. We utilize a semiclassical approach which has recently been developed. In this approach, the non-Abelian nature of the spin diffusion along a classical trajectory plays a crucial role. New analytical expressions with one crossover parameter are semiclassically derived for the average conductance, conductance variance, and shot noise. Moreover numerical results on a random-matrix model describing the crossover from the Gaussian orthogonal ensemble to the Gaussian symplectic ensemble are compared with the semiclassical expressions.

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I. INTRODUCTION

A chaotic quantum transport of an electron in a cavity is caused by either implanted impurities or bumpy boundaries and provides directly measurable *quantum signatures of chaos*,[1] such as the conductance variance. Universal aspects of a chaotic transport have been investigated by means of the random-matrix theory (RMT).[2] In the RMT, quantum systems are classified into symmetry classes. A chaotic system with time-reversal symmetry is described by the Gaussian orthogonal ensemble (GOE). When the time-reversal symmetry is broken by applying a magnetic field, the Gaussian unitary ensemble (GUE) becomes a suitable model. If a system with time-reversal symmetry has a spin-orbit interaction, one needs to employ the Gaussian symplectic ensemble (GSE).

We consider the case that two leads are attached to a cavity and the number of the lead channels are N_1 and N_2. An electron transport in the cavity is described by the scattering matrix.[3,4] Replacing the scattering matrix by a random matrix, the RMT phenomenologically predicts the average conductance G, conductance variance Var G, and shot noise P at zero temperature as[5,6]

$$\frac{G}{G_0}=\frac{2N_1N_2}{N-1+\dfrac{2}{\beta}}=\frac{2N_1N_2}{N}\left\{1+\frac{1-\dfrac{2}{\beta}}{N}+\left(\frac{1-\dfrac{2}{\beta}}{N}\right)^2\right\}+O\left(\frac{1}{N^2}\right), \tag{1}$$

$$\frac{\mathrm{Var}\,G}{G_0^2}=\frac{8N_1\left(N_1-1+\dfrac{2}{\beta}\right)N_2\left(N_2-1+\dfrac{2}{\beta}\right)}{\beta\left(N-2+\dfrac{2}{\beta}\right)\left(N-1+\dfrac{2}{\beta}\right)^2\left(N-1+\dfrac{4}{\beta}\right)}=\frac{8N_1^2N_2^2}{\beta N^4}+O\left(\frac{1}{N}\right), \tag{2}$$

$$\frac{P}{P_0}=\frac{2N_1\left(N_1-1+\dfrac{2}{\beta}\right)N_2\left(N_2-1+\dfrac{2}{\beta}\right)}{\left(N-2+\dfrac{2}{\beta}\right)\left(N-1+\dfrac{2}{\beta}\right)\left(N-1+\dfrac{4}{\beta}\right)}=\frac{2N_1^2N_2^2}{N^3}+\left(\frac{4}{\beta}-2\right)\frac{N_1N_2}{N^4}(N_1-N_2)^2+O\left(\frac{1}{N}\right) \tag{3}$$

with $N=N_1+N_2$. Here $\beta=1$, 2, and 4 correspond to the GOE, GUE, and GSE symmetry classes, respectively. These expressions include the contributions from the spin degrees of freedom.[7] The constants G_0 and P_0 are $G_0=e^2/(\pi\hbar)$ and $P_0=2e^3|V|/(\pi\hbar)$, respectively, where e is the unit electric charge and V is the bias voltage. If N_1 is equal to N_2 and very large, the leading term of the shot noise is insensitive to a change in the symmetry.

When a very weak magnetic field is applied to the cavity, the time-reversal symmetry is only partially broken. In this case, a crossover from the GOE to GUE is observed. This GOE-GUE crossover regime can also be analyzed by a parametric RMT model, and analytic predictions describing a chaotic quantum transport are known.[8] Recently, a chaotic transport in the GSE-GUE crossover regime was also studied within the RMT framework.[9] In this regime, a very weak magnetic field breaks the time-reversal symmetry of a system with a spin-orbit interaction.

The aim of this paper is to study another case, the crossover from the GOE to GSE, in which the system has a very weak spin-orbit interaction preserving the time-reversal symmetry. In the experimental point of view, the GOE-GSE crossover can be realized, if the spin-orbit interaction (or Aharonov-Casher effect) is controlled by applying an electric field in a chaotic conductor.[10,11] In this case, a parametric RMT model is also known,[12,13] and the diagrammatic perturbation theory has been used to evaluate some transport properties.[14–17] Here we employ a semiclassical approach which has recently been developed.[18–23] In a semiclassical evaluation, the transmission amplitude is treated by the pathintegral method, where all the classical paths must, in principle, be taken into account. However recent studies clarified that almost the same but partially time reversed pairs of classical trajectories contributed to the conductance[24–27] so that the calculation was greatly simplified. Then it was shown that the semiclassical approach could precisely reproduce the RMT predictions in Eqs. (1)–(3).[21–23] Moreover, when a similar approach is applied to the parametric spectral correlations in the GOE-GUE, GUE-GUE, GOE-GOE, and GSE-GSE regimes, it can also reproduce the RMT predictions.[28–31]

Thus the semiclassical approach has become a practical tool to find a new prediction, even if the RMT analysis is difficult. As this approach was already applied to the parametric spectral correlations in the GOE-GSE crossover regime,[31] we naturally expect that it can be used in the analysis of a transport.

Considering the non-Abelian nature of the spin diffusion along the classical trajectories, we extend the semiclassical technique to derive analytic expressions for the transport properties. Our results on the average conductance, conductance variance, and shot noise are given in Eqs. (25), (31), and (35). The crossover from the GOE to GSE is controlled by one parameter depending on the diffusion constant of the spin. The GOE and GSE results are reproduced in the limiting cases of the parameter.

This paper is organized in the following way. In Sec. II, a semiclassical expression of the transmission amplitude is presented. We put a stress on the statistical aspects of the expression. In Sec. III, using the semiclassical expression, we calculate the average conductance, conductance variance, and shot noise. In Sec. IV, these results are compared with numerical calculations on a random-matrix model. We finally summarize the paper in Sec. V.

II. SEMICLASSICAL EXPRESSION OF THE TRANSMISSION AMPLITUDE

The semiclassical theory employs the transmission amplitude t_{a_1,a_2}, which represents the propagator of a wave packet from the channel a_1 in one lead to a_2 in another lead. Bolte and Keppeler derived a semiclassical expression of the transmission amplitude with spin variables[32]

$$t_{a_1,a_2} \sim \sqrt{\frac{2}{T_H}} \sum_{\alpha:a_1\to a_2} \mathcal{A}_\alpha \Delta_\alpha e^{iS_\alpha/\hbar}, \tag{4}$$

where T_H is the Heisenberg time $T_H=\frac{\Omega(E)}{(2\pi\hbar)^{f-1}}$. Here $\Omega(E)$ is the phase volume density including spin degrees of freedom at the energy E and f is the spacial dimension. Throughout this paper, we study the two-dimensional case $f=2$. Two leads are assumed to have N_1 and N_2 channels, i.e., $a_1=1,2,\ldots,N_1$ and $a_2=1,2,\ldots,N_2$. The classical action of the orbit α is $S_\alpha=\int_\alpha \boldsymbol{p}\cdot d\boldsymbol{q}$, where $\boldsymbol{q}$ and $\boldsymbol{p}$ are the position and momentum variables.

The stability amplitude is decoupled into two factors $\mathcal{A}_\alpha$ and Δ_α. The first factor $\mathcal{A}_\alpha$ accounts for the stability in the position and momentum space and the second factor Δ_α originates from the spin dynamics. Both $\mathcal{A}_\alpha$ and Δ_α are uniquely determined, when the classical trajectory α governed by the microscopic Hamiltonian is specified. However their statistical behavior is independent of the details of the trajectory. As discussed in Refs. 21, 22, and 33, the stability amplitude $\mathcal{A}_\alpha$ is related to the survival probability in the chaotic cavity. Postulating an ergodic motion in the position and momentum space, we obtain the following sum rule:

$$\sum_{\alpha:a_1\to a_2} |\mathcal{A}_\alpha|^2 = \int_0^\infty dT e^{-(2N/T_H)T}, \tag{5}$$

where $T_H/2N$ ($N=N_1+N_2$) can be regarded as the dwell time inside the cavity so that the inverse is the escape rate. The escape rate is related to the position and momentum variables and is unrelated to the spin variables. Hence the dwell time should be $T_H/2N$ rather than T_H/N. The spin matrix Δ_α is defined as

$$\Delta(t) = e^{i\phi(t)\sigma_z/2} e^{i\theta(t)\sigma_x/2} e^{i\psi(t)\sigma_z/2} \tag{6}$$

along a trajectory α, where $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ consists of the Pauli matrices. The time evolution of the Euler angles $[\psi(t),\theta(t),\phi(t)]$ is microscopically determined by the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Delta(t) = \mathcal{H}\Delta(t), \tag{7}$$

where $\mathcal{H}$ is the effective Hamiltonian which describes the spin-orbit interaction. We assume that the spin dynamics is subdominant in the semiclassical limit. That is, the dynamics of the position and momentum variables are determined by the spacial Hamiltonian without spin degrees of freedom while the spin is influenced by the momentum motion via the spin-orbit interaction. The effective Hamiltonian in Eq. (7) describes such a subdominant dynamics of the spin variables. A similar hierarchical structure has successfully been employed to analyze the GOE-GUE crossover regime:[22,28–30] the resulting physical quantities are in agreement with the corresponding RMT expressions. An RMT prediction for the parametric spectral form factor in the GSE symmetry class was also reproduced in a similar way.[31]

Since bumpy boundaries of a cavity induce a chaotic behavior in the position and momentum variables, the momentum effectively plays a role of a stochastic magnetic field.[31] Then the effective Hamiltonian which describes the time evolution of the spin can be written as

$$\mathcal{H} = \gamma_{\rm so}\boldsymbol{h}\cdot\left(\frac{\hbar}{2}\boldsymbol{\sigma}\right), \tag{8}$$

where $\gamma_{\rm so}$ is the coupling constant and $\boldsymbol{h}=[h_x(t),h_y(t),h_z(t)]$ is the effective stochastic magnetic field. We assume that the classical spin undergoes a Brownian motion on the Bloch sphere[34] due to the stochastic magnetic field satisfying

$$\langle\langle h_\alpha(t)h_{\alpha'}(t')\rangle\rangle = 2\mathcal{D}\delta_{\alpha,\alpha'}\delta(t-t'), \quad \alpha,\alpha'=x,y,z, \tag{9}$$

where the brackets $\langle\langle\cdots\rangle\rangle$ denote an average over the stochastic process of the magnetic field and $\mathcal{D}$ is the diffusion constant. Then the probability density function of the Euler angle $P(\psi,\theta,\phi)$ (with the measure $\sin\theta d\psi d\theta d\phi$) obeys the Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \gamma_{\rm so}^2 \mathcal{D}\mathcal{L}P, \tag{10}$$

where $\mathcal{L}$ is the Laplace-Beltrami operator

$$\mathcal{L} = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\left(\frac{\partial^2}{\partial\psi^2} + \frac{\partial^2}{\partial\phi^2} - 2\cos\theta\frac{\partial^2}{\partial\psi\,\partial\phi}\right). \tag{11}$$

This stochastic dynamics can exactly be analyzed by using Wigner's D function.[35] Let us suppose that the initial Euler angles at $t=0$ are $\omega'=(\psi',\theta',\phi')$. Then the conditional probability to find the angles at $\omega=(\psi,\theta,\phi)$ after time t is

$$g(\omega;t|\omega') = \sum_{j=0}^{\infty}\sum_{m=-j}^{j}\sum_{n=-j}^{j}\frac{2j+1}{32\pi^2}D^j_{m,n}(\psi,\theta,\phi) \times\{D^j_{m,n}(\psi',\theta',\phi')\}^* e^{-j(j+1)\gamma_{\rm so}^2\mathcal{D}t}. \tag{12}$$

Here an asterisk signifies a complex conjugate and Wigner's D function is defined as

$$D^j_{m,n}(\psi,\theta,\phi) = e^{im\phi}d^j_{m,n}(\theta)e^{in\psi}, \tag{13}$$

where

$$d^j_{m,n}(\theta) = \sqrt{\frac{(j+m)!(j-m)!}{(j+n)!(j-n)!}} \times\cos^{m+n}(\theta/2)\sin^{m-n}(\theta/2)P^{(m-n,m+n)}_{j-m}(\cos\theta) \tag{14}$$

in terms of the Jacobi polynomials $P^{(a,b)}_k(x)$. The index j is an integer or a half odd integer ($j=0,1/2,1,3/2,\ldots$ and $m,n=-j,-j+1,\ldots,j$). The conditional probability $g(\omega;t|\omega')$ satisfies a normalization condition

$$\int d\omega g(\omega;t|\omega') = \int_0^{\pi}d\theta\int_0^{4\pi}d\phi\int_0^{4\pi}d\psi\sin\theta g(\omega;t|\omega') = 1.$$

III. TRANSPORT IN THE GOE-GSE CROSSOVER REGIME

A. Average conductance

The average conductance G is written in terms of the transmission amplitude t_{a_1,a_2} as

$$\frac{G}{G_0} = \langle\mathrm{Tr}(\mathbf{t}\mathbf{t}^\dagger)\rangle = \sum_{a_1=1}^{N1}\sum_{a_2=1}^{N_2}\mathrm{Tr}\{t_{a_1,a_2}(t^\dagger)_{a_2,a_1}\}. \tag{15}$$

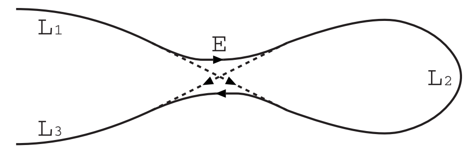

FIG. 1. The RS pair. The solid and dashed curves are, respectively, α and γ orbits in the text.

Here the transmission matrix $\mathbf{t}$ is a $2N_1\times 2N_2$ matrix which consists of the 2×2 blocks t_{a_1,a_2}. Then a semiclassical expression

$$\langle\mathrm{Tr}(\mathbf{t}\mathbf{t}^\dagger)\rangle = \frac{2}{T_H}\left\langle\sum_{a_1,a_2}\sum_{\alpha,\gamma:a_1\to a_2}\mathcal{A}_\alpha\mathcal{A}_\gamma^*\langle\langle\mathrm{Tr}(\Delta_\alpha\Delta_\gamma^\dagger)\rangle\rangle e^{i/\hbar(S_\alpha-S_\gamma)}\right\rangle \tag{16}$$

follows from Eq. (4). Here the brackets $\langle\cdots\rangle$ mean an energy average, which eliminates the fluctuations of the physical quantities. If the difference between the actions S_α and S_γ is sufficiently large, the exponential term $e^{i/\hbar(S_\alpha-S_\gamma)}$ rapidly oscillates in the semiclassical limit $\hbar\to 0$, which eventually vanishes after averaging. Hence, in order to give a finite contribution, the trajectories α and γ are mutually almost the same. Then the identical trajectories $\alpha=\gamma$ yield the first-order approximation, which is referred to as "the diagonal approximation."[36] These mutually identical trajectory pairs yield the following contribution:

$$\langle\mathrm{Tr}(\mathbf{t}\mathbf{t}^\dagger)\rangle_1 = \frac{2}{T_H}\sum_{a_1,a_2}\sum_{\alpha}\mathrm{Tr}(\Delta_\alpha\Delta_\alpha^\dagger)|\mathcal{A}_\alpha|^2 = \frac{4}{T_H}\sum_{a_1,a_2}\sum_{\alpha}|\mathcal{A}_\alpha|^2 = \frac{4}{T_H}N_1N_2\int_0^\infty dTe^{-(2N/T_H)T} = \frac{2N_1N_2}{N}. \tag{17}$$

Here we used the sum rule Eq. (5) for the stability amplitude $\mathcal{A}_\alpha$. In this calculation, a product of the spin matrices is reduced to the identity matrix and the trace yields a factor 2. The diagonal approximation does not discriminate the symmetry classes.

The second-order approximation comes from the Richter-Sieber (RS) pairs[26,27] drawn in Fig. 1. In the RS pair, two trajectories come close to each other in the encounter region and go in the opposite directions on one loop. We can symbolically write RS pairs (see Fig. 1) as

$$\alpha{:}L_1EL_2\bar{E}L_3,$$

$$\gamma{:}L_1E\bar{L}_2\bar{E}L_3,$$

where E implies one of the two trajectory segments in the encounter region where two loops are connected and $\bar{E}$ implies the time reverse of E. The loops are denoted as L_1, L_2, and L_3, respectively, and $\bar{L}_j$ ($j=1,2,3$) is the time reverse of L_j. Using these notations, we can write the RS pair contribution as

$$\langle \mathrm{Tr}(\mathbf{tt}^\dagger)\rangle_2 = \frac{2}{T_H}\sum_{a_1,a_2}\sum_{\substack{\alpha:L_1EL_2\bar{E}L_3\\ \gamma:L_1E\bar{L}_2\bar{E}L_3}} \langle \mathcal{A}_\alpha\mathcal{A}_\gamma^*\langle\langle \mathrm{Tr}(\Delta_\alpha\Delta_\gamma^\dagger)\rangle\rangle e^{(i/\hbar)\Delta S}\rangle, \tag{18}$$

where ΔS is the action difference $S_\alpha - S_\gamma$. The spin matrices Δ_α and Δ_γ are factored into the loop and encounter parts as

$$\Delta_\alpha = \Delta_{L_1}\Delta_E\Delta_{L_2}(\Delta_E)^{-1}\Delta_{L_3}, \tag{19}$$

$$\Delta_\gamma = \Delta_{L_1}\Delta_E(\Delta_{L_2})^{-1}(\Delta_E)^{-1}\Delta_{L_3}. \tag{20}$$

Along the trajectories the non-Abelian nature of the spin operators must be taken into account. A time-reversal operation of a spin matrix is realized by a matrix inversion.

We divide the whole time elapsed on a trajectory into the loop and encounter parts, i.e., T_1, T_2, and T_3 for L_1, L_2, and L_3, respectively, and t_{enc} for E. It should be noted that the existence of the encounter affects the survival probability $e^{-(2N/T_H)T}$ in Eq. (5):[21,22] if one of the two orbit segments in the encounter is inside the cavity, the other segment must also remain inside. Hence the survival probability is modified into $e^{-(2N/T_H)(T-t_{\mathrm{enc}})}$.

In the encounter region, the classical actions of the two trajectories are slightly different. The action difference can be estimated by using the coordinates (s,u) along the stable and unstable manifolds within the ranges $s,u \in [-c,c]$.[21,22] The time duration t_{enc} inside the encounter region is related to the Lyapunov exponent λ as $t_{\mathrm{enc}} \sim \frac{1}{\lambda}\ln\frac{c^2}{|us|}$, and the action difference is expressed as

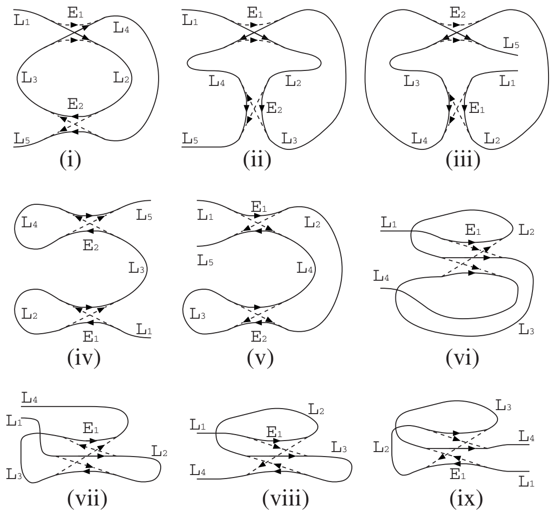

FIG. 2. The diagrams of the third-order contributing to the conductance.

$$\Delta S = us. \tag{21}$$

The number density of encounters in a trajectory with an elapsed time $T = T_1 + T_2 + T_3 + 2t_{\mathrm{enc}}$ is evaluated as[21,22]

$$\omega(s,u)dsdu = \int_{\substack{T_1,T_2>0\\ T_1+T_2<T-2t_{\mathrm{enc}}}} dT_1 dT_2 \frac{1}{t_{\mathrm{enc}}\Omega/2} dsdu \tag{22}$$

by taking account of T_1, $T_2 > 0$, and $T_3 = T - T_1 - T_2 - 2t_{\mathrm{enc}} > 0$.

On the other hand, the spin-diffusion term is calculated as

$$\begin{aligned}\langle\langle \mathrm{Tr}(\Delta_\alpha\Delta_\gamma^\dagger)\rangle\rangle &= \int d\omega_{L_1}d\omega_{L_2}d\omega_{L_3}d\omega_{E_1}\mathrm{Tr}\{(\Delta_{L_2})^2\}g(\omega_{L_1},T_1|0,0,0)g(\omega_{L_2},T_2|0,0,0)g(\omega_{L_3},T_3|0,0,0)g(\omega_{L_E},t_{\mathrm{enc}}|0,0,0)\\ &= -1 + 3e^{-2\gamma_{\mathrm{so}}^2\mathcal{D}T_2}.\end{aligned} \tag{23}$$

Using the above formulas, we find that the RS contribution is

$$\begin{aligned}\langle \mathrm{Tr}(\mathbf{tt}^\dagger)\rangle_2 &= \frac{2N_1N_2}{T_H}\int_{-c}^{c} dsdu \int_{t_{\mathrm{enc}}}^{\infty} dT\omega(s,u)e^{-(2N/T_H)(T-t_{\mathrm{enc}})}\langle\langle \mathrm{Tr}(\Delta_\alpha\Delta_\gamma^\dagger)\rangle\rangle\\ &= \frac{2N_1N_2}{T_H}\int_0^\infty dT_1dT_2dT_3\int_{-c}^{c} dsdu \frac{2e^{isu/\hbar}}{\Omega t_{\mathrm{enc}}} e^{-(2N/T_H)(T_1+T_2+T_3+t_{\mathrm{enc}})}(3e^{-2\gamma_{\mathrm{so}}^2\mathcal{D}T_2}-1) = \frac{N_1N_2}{(N_1+N_2)^2}\left(1-\frac{3}{1+\gamma_{\mathrm{so}}^2\mathcal{D}T_H/N}\right).\end{aligned} \tag{24}$$

The last line of the above formula is obtained by the following criterion: after expanding the formula in t_{enc} and integrating each term over (s,u), any term dependent on t_{enc} vanishes in the semiclassical limit, and a finite contribution comes only from the terms independent of t_{enc}.

The third-order contribution to the average conductance comes from the diagrams drawn in Fig. 2. In the Appendix, the spin-diffusion terms are listed. Using these results, we arrive at an expression of the average conductance

$$\frac{G}{G_0} = \frac{2N_1N_2}{N} + \frac{N_1N_2}{N^2}\left(1 - \frac{3}{1+\xi}\right) + \frac{N_1N_2}{2N^3}\left\{1 - \frac{3}{1+\xi} + \frac{3}{(1+\xi)^2} + \frac{3}{(1+\xi)^3}\right\} + O(1/N^2), \tag{25}$$

where ξ is the crossover parameter defined as $\xi = \gamma_{so}^2 \mathcal{D} T_H / N$. We can easily check that Eq. (25) reproduces the $1/N$ expansions of the GOE and GSE formulas (1) by taking the limits $\xi \to 0$ and $\xi \to \infty$, respectively. It follows from this result that only one parameter ξ is necessary to describe the GOE-GSE crossover.

B. Conductance variance

The conductance variance Var G is written in terms of the transmission matrix $\mathbf{t}$ as

$$\frac{\mathrm{Var}\, G}{G_0^2} = \langle \{\mathrm{Tr}(\mathbf{tt}^\dagger)\}^2 \rangle - \langle \mathrm{Tr}(\mathbf{tt}^\dagger)\rangle^2. \tag{26}$$

The semiclassical expression of the first term is

$$\langle \{\mathrm{Tr}(tt^\dagger)\}^2\rangle = \frac{4}{T_H^2}\sum_{a_1,a_2}\sum_{c_1,c_2}\sum_{\alpha,\beta:a_1\to a_2}\sum_{\gamma,\delta:c_1\to c_2}\langle \mathcal{A}_\alpha \mathcal{A}_\beta^* \mathcal{A}_\gamma \mathcal{A}_\delta^* e^{i(S_\alpha - S_\beta + S_\gamma - S_\delta)/\hbar} \langle\langle \mathrm{Tr}(\Delta_\alpha \Delta_\beta^\dagger)\mathrm{Tr}(\Delta_\gamma \Delta_\delta^\dagger)\rangle\rangle\rangle. \tag{27}$$

Let us first adopt the diagonal approximation, for which we have two types of contributions. One is given by setting $\alpha = \beta$ and $\gamma = \delta$, where a_1, a_2, c_1, and c_2 are all independent. The other choice is $\alpha = \delta$, $\gamma = \beta$ and $a_1 = c_1$, $a_2 = c_2$. The numbers of possible channel combinations are $N_1^2 N_2^2$ and $N_1 N_2$, respectively. These contributions are summed up to yield

$$\langle \{\mathrm{Tr}(\mathbf{tt}^\dagger)\}^2\rangle_1 = \frac{4}{T_H^2}\left\langle 4\sum_{\substack{a_1,c_1\\ a_2,c_2}}\sum_{\substack{\alpha=\beta:a_1\to a_2\\ \gamma=\delta:c_1\to c_2}} |\mathcal{A}_\alpha|^2|\mathcal{A}_\gamma|^2 + \sum_{\substack{a_1=c_1\\ a_2=c_2}}\sum_{\substack{\alpha=\delta:a_1\to a_2\\ \gamma=\beta:a_1\to a_2}} |\mathcal{A}_\alpha|^2|\mathcal{A}_\gamma|^2\langle\langle |\mathrm{Tr}(\Delta_\alpha\Delta_\gamma^\dagger)|^2\rangle\rangle\right\rangle. \tag{28}$$

Here the spin-diffusion term is

$$\langle\langle |\mathrm{Tr}(\Delta_\alpha \Delta_\gamma^\dagger)|^2\rangle\rangle = \int d\omega_\alpha d\omega_\gamma g(\omega_\alpha, T_\alpha|0,0,0) g(\omega_\gamma, T_\gamma|0,0,0) = 1 + 3e^{-2\gamma_{so}^2 \mathcal{D}(T_\alpha + T_\gamma)}, \tag{29}$$

where T_α and T_γ are the times elapsed on the trajectories α and γ, respectively. Using this expression, one can obtain the diagonal contribution

$$\langle \{\mathrm{Tr}(\mathbf{tt}^\dagger)\}^2\rangle_1 = \frac{4N_1^2N_2^2}{N^2} + \frac{N_1N_2}{N^2}\left\{1 + \frac{3}{(1+\xi)^2}\right\}. \tag{30}$$

To go beyond the diagram approximation, we note that the next order diagrams are classified into d families and x families as shown in Fig. 3:[22] d quadruplets are drawn in the diagrams (i)–(vii) while x quadruplets in (viii).

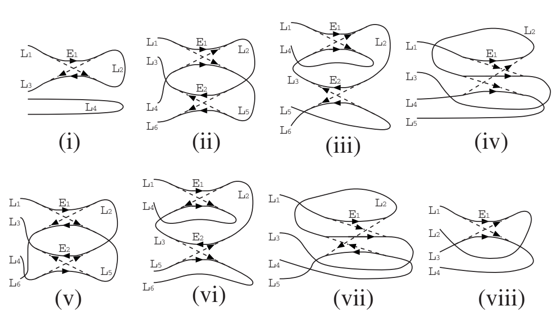

FIG. 3. The diagrams contributing to the conductance variance.

Let us write the next order term as

$$\langle \{\mathrm{Tr}(\mathbf{tt}^\dagger)\}^2\rangle_2 = (N_1^2N_2^2 + N_1N_2)\left(\frac{d_1}{N^3} + \frac{d_2}{N^4}\right) + N_1N_2\frac{x_1}{N^2}.$$

Here the coefficients d_1, d_2, and x_1 are obtained from the families of quadruplets. The coefficient d_1 comes from the diagram (i): quadruplets consisting of one diagonal pair and one RS pair. On the other hand, many diagrams have to be taken into account to calculate d_2, i.e., (1): quadruplets consisting of one diagonal pair and one pair contributing to the $O(1/N)$ term in the expansion (25) of the average conductance, (2): two RS pairs, and (3): the diagrams (ii)–(vii) shown in Fig. 3. The coefficient x_1 is calculated from the diagram (viii) in Fig. 3. Considering the contributions from these diagrams, we obtain

$$d_1 = 4 - \frac{12}{1+\xi},$$

$$d_2 = 5 - \frac{12}{1+\xi} + \frac{21}{(1+\xi)^2} + \frac{6}{(1+\xi)^3},$$

$$x_1 = -1 - \frac{3}{(1+\xi)^2}.$$

Then we find the expression of the conductance variance for $N_1 \gg 1$ and $N_2 \gg 1$ as

$$\frac{\mathrm{Var}\, G}{G_0^2} = \frac{N_1^2 N_2^2}{N^4}\left\{2 + \frac{6}{(1+\xi)^2}\right\} + O\left(\frac{1}{N}\right). \tag{31}$$

One can check that the GOE and GSE limits agree with Eq. (2).

C. Shot noise

The shot noise P is written in the form

$$\frac{P}{P_0} = \langle \mathrm{Tr}(\mathbf{tt}^\dagger - \mathbf{tt}^\dagger\mathbf{tt}^\dagger)\rangle. \tag{32}$$

The second term is semiclassically expressed as

$$\langle \mathrm{Tr}(\mathbf{tt}^\dagger\mathbf{tt}^\dagger)\rangle = \frac{4}{T_H^2}\sum_{\substack{a_1,a_2\\ c_1,c_2}}\sum_{\substack{\alpha:a_1\to a_2,\beta:c_1\to a_2\\ \gamma:c_1\to c_2,\delta:a_1\to c_2}} \times \langle A_\alpha A_\beta^* A_\gamma A_\delta^* e^{i/\hbar(S_\alpha - S_\beta + S_\gamma - S_\delta)} \times \langle\langle \mathrm{Tr}(\Delta_\alpha \Delta_\beta^\dagger \Delta_\gamma \Delta_\delta^\dagger)\rangle\rangle\rangle. \tag{33}$$

The diagonal contribution consists of two terms. One term has $\alpha=\beta$ and $\gamma=\delta$ where we need to set $a_1=c_1$. The other term has $\alpha=\delta$, $\beta=\gamma$, and $a_2=c_2$. We sum up these two terms and obtain

$$\langle \mathrm{Tr}(\mathbf{tt}^\dagger\mathbf{tt}^\dagger)\rangle_1 = \frac{16}{T_H^2}\left\langle \sum_{a_1,a_2,c_2}\sum_{\substack{\alpha:a_1\to a_2\\ \gamma:a_1\to c_2}} |A_\alpha|^2|A_\gamma|^2 \langle\langle \mathrm{Tr}(\Delta_\alpha\Delta_\alpha^\dagger\Delta_\gamma\Delta_\gamma^\dagger)\rangle\rangle + \sum_{a_1,a_2,c_1}\sum_{\substack{\alpha:a_1\to a_2\\ \beta:c_1\to a_2}} |A_\alpha|^2|A_\beta|^2 \langle\langle \mathrm{Tr}(\Delta_\alpha\Delta_\beta^\dagger\Delta_\beta\Delta_\alpha^\dagger)\rangle\rangle \right\rangle = 2\frac{N_1N_2}{N}. \tag{34}$$

It follows that the diagonal contributions to $\langle \mathrm{Tr}(\mathbf{tt}^\dagger)\rangle$ and $\langle \mathrm{Tr}(\mathbf{tt}^\dagger\mathbf{tt}^\dagger)\rangle$ are both $2\frac{N_1N_2}{N}$, and mutually cancel.

Hence the RS pair contribution to $\mathrm{Tr}(\mathbf{tt}^\dagger)$ and the contribution to $\mathrm{Tr}(\mathbf{tt}^\dagger\mathbf{tt}^\dagger)$ from the quadruplets drawn in Fig. 3 (i) and Fig. 3 (viii) have to be calculated. Moreover we take account of the additional diagrams shown in Fig. 4.[21,22] Summing up these contributions, we finally obtain the shot noise in the crossover regime as

$$\frac{P}{P_0} = 2\frac{N_1^2N_2^2}{N^3} + \frac{N_1N_2(N_1-N_2)^2}{N^4}\left(\frac{3}{1+\xi} - 1\right) + O(1/N). \tag{35}$$

Let us denote the $O(1)$ terms of G and P by δG and δP, respectively. It can be seen from Eqs. (25) and (35) that

$$\frac{\delta P/P_0}{\delta G/G_0} = -\left(\frac{N_1-N_2}{N_1+N_2}\right)^2, \tag{36}$$

which is a universal relation established in Ref. 17.

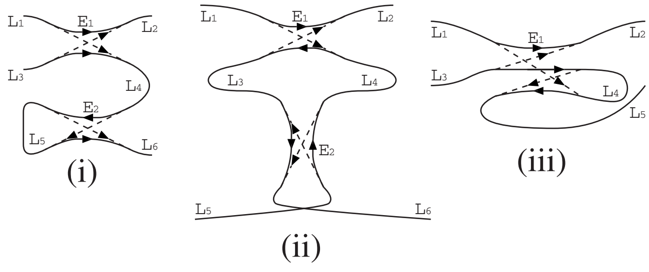

FIG. 4. The diagrams contributing to the shot noise.

IV. COMPARISON WITH A RANDOM-MATRIX MODEL

In this section, a random-matrix model on the GOE-GSE crossover is numerically analyzed and the results are compared with the semiclassical formulas. In the theory of random matrices, a time-reversal invariant quantum Hamiltonian with spin 1/2 is simulated by a self-dual real quaternion random matrix.[37] A real quaternion q is a linear combination

$$q = q_0e_0 + q_1e_1 + q_2e_2 + q_3e_3, \tag{37}$$

where q_j are real numbers and called the jth component of q. The bases e_0,e_1,e_2,e_3 can be represented by 2×2 matrices as

$$e_0 \to \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \quad e_1 \to \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix},$$

$$e_2 \to \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \quad e_3 \to \begin{pmatrix}0 & i\\ i & 0\end{pmatrix} \tag{38}$$

so that q_0e_0 is equated with a real number q_0. The dual quaternion of q is defined as

$$\bar{q} = q_0e_0 - q_1e_1 - q_2e_2 - q_3e_3. \tag{39}$$

When an $m\times n$ real quaternion matrix Q has the (j,l) element q_{jl}, we define that the $n\times m$ dual matrix $\bar{Q}$ has the (j,l) element $\bar{q}_{lj}$. If a square real quaternion matrix satisfies $Q=\bar{Q}$, then Q is called a self-dual real quaternion matrix.

The parametric motion of a self-dual real quaternion random matrix is realized in the framework of Dyson's matrix Brownian motion model.[38] It is postulated that the probability density function (pdf) of an $M\times M$ self-dual real quaternion matrix H is

$$P(H;\tau|R)dH \propto \exp\left\{-2\frac{\mathrm{Tr}(H-e^{-\tau}R)^2}{1-e^{-2\tau}}\right\}dH \tag{40}$$

with

$$dH = \prod_{j=1}^{M} dH_{jj}\prod_{j<l}^{M}\prod_{k=0}^{3} dH_{jl}^{(k)}. \tag{41}$$

Here $H_{jl}^{(k)}$ is the kth component of H_{jl}. We are interested in the parametric motion of the matrix H depending on the fictitious time parameter τ.

At the initial time $\tau=0$, this pdf is reduced to

$$P(H;0|R)dH = \delta(H-R)dH \tag{42}$$

so that the self-dual real quaternion matrix R gives the initial condition of the parametric motion. On the other hand, in the limit $\tau\to\infty$,

$$P(H;\infty|R)dH \propto \exp(-2\ \mathrm{Tr}\ H^2)dH, \tag{43}$$

which is the pdf of the GSE.

Let us suppose that the elements of the initial matrix R have only the zeroth components (R is then a real symmetric matrix) and that the pdf of R is

$$P_{\mathrm{GOE}}(R)dR \propto \exp\left(-\frac{1}{2}\mathrm{Tr}\ R^2\right)dR \tag{44}$$

with

$$dR = \prod_{j=1}^{M} dR_{jj}\prod_{j<l}^{M} dR_{jl}, \tag{45}$$

which is the pdf of the GOE. Then we can calculate the pdf of H at a fictitious time τ as

$$\begin{aligned} P(H)dH &= \left\{\int dRP(H;\tau|R)P_{\mathrm{GOE}}(R)\right\}dH \propto \left[\int dR\ \exp\left\{-2\frac{\mathrm{Tr}(H-e^{-\tau}R)^2}{1-e^{-2\tau}} - \frac{1}{2}\mathrm{Tr}\ R^2\right\}\right]dH \\ &\propto \exp\left[-\frac{2}{1+3e^{-2\tau}}\sum_{j=1}^{M}(H_{jj}^{(0)})^2 - \frac{4}{1+3e^{-2\tau}}\sum_{j<l}^{M}(H_{jl}^{(0)})^2 - \frac{4}{1-e^{-2\tau}}\sum_{j<l}^{M}\{(H_{jl}^{(1)})^2+(H_{jl}^{(2)})^2+(H_{jl}^{(3)})^2\}\right]dH, \end{aligned} \tag{46}$$

which describes the crossover from the GOE (at $\tau=0$) to GSE (at $\tau=\infty$). The components of the elements H_{jl} ($j\le l$) are independently distributed according to Gaussian density functions.

If an $M\times M$ real quaternion matrix Z satisfies

$$Z\bar{Z} = \bar{Z}Z = I_M \tag{47}$$

(I_M is the $M\times M$ identity matrix), then Z is called a symplectic matrix. If the elements of a symplectic matrix U only have the zeroth elements, then U is a real orthogonal matrix. It is known that the measure dH is invariant under the symplectic transformation $H\mapsto\bar{Z}HZ$ and the measure dS is invariant under the orthogonal transformation $S\mapsto U^{\mathrm{T}}SU$ (U^{T} is the transpose of U). It follows that the pdf $P(H)dH$ in Eq. (46) is invariant under the orthogonal transformation $H\mapsto U^{\mathrm{T}}HU$.

We go back to the problem of a chaotic cavity with M bound states to which two leads with N_1 and N_2 propagating modes are attached ($M\ge N=N_1+N_2$). We are interested in the limit $M\to\infty$. Let us suppose that the $M\times M$ matrix H describing the scattering in the cavity is a random matrix distributed according to the crossover pdf in Eq. (46). Then the $N\times N$ scattering matrix S is[2]

$$S = I_N + i2\pi W^{\mathrm{T}}(H-E_F-i\pi WW^{\mathrm{T}})^{-1}W, \tag{48}$$

where E_F is the Fermi energy and the elements of an $M\times N$ real matrix W are the coupling constants between the cavity and the leads.

Assuming that the tunnel probability of the leads are 1, we can see that the eigenvalues of $W^{\mathrm{T}}W$ are all $M/(\rho\pi^2)$, where ρ is the eigenvalue density of H at the Fermi energy. Then a singular value decomposition

$$W = UDV \tag{49}$$

holds, where U and V are $M\times M$ and $N\times N$ real orthogonal matrices, respectively, and D is an $M\times N$ matrix

$$D = \frac{1}{\pi}\sqrt{\frac{M}{\rho}}\tilde{W} \tag{50}$$

with

$$\tilde{W} = \begin{pmatrix} I_N \\ O \end{pmatrix}. \tag{51}$$

Here O is an $(M-N)\times N$ matrix consisting of zero elements. When the Fermi energy E_F is set to zero (so that $\rho=\sqrt{2M}/\pi$), the scattering matrix S can be rewritten as

$$S = I_N + i\sqrt{2M}\widetilde{W}^{\mathrm{T}}\left(\widetilde{H} - i\sqrt{\frac{M}{2}}\widetilde{W}\widetilde{W}^{\mathrm{T}}\right)^{-1}\widetilde{W}, \qquad (52)$$

where

$$\widetilde{H} = (U\widetilde{V})^{\mathrm{T}}H(U\widetilde{V}) \qquad (53)$$

with

$$\widetilde{V} = \begin{pmatrix} V & O \\ O & I_{M-N} \end{pmatrix}. \qquad (54)$$

Since $U\widetilde{V}$ is a real orthogonal matrix, the $M \times M$ matrix $\widetilde{H}$ is also distributed according to the pdf in Eq. (46) (with H replaced by $\widetilde{H}$).

Thus the scattering matrix S can numerically be generated by using the Gaussian pdf Eq. (46) of $\widetilde{H}$ and the relation in Eq. (52). Replacing the quaternion elements of S by the 2×2 matrix representations in Eq. (38), we obtain a $2N \times 2N$ matrix $\widetilde{S}$. It is written in terms of the $2N_1 \times 2N_2$ transmission matrix $\mathbf{t}$ as

$$\widetilde{S} = \begin{pmatrix} \mathbf{r} & \mathbf{t}^{\dagger} \\ \mathbf{t} & \mathbf{r}' \end{pmatrix}, \qquad (55)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the reflection matrices. Then the average conductance G can be evaluated as

$$\frac{G}{G_0} = \langle \mathrm{Tr}(\mathbf{t}\mathbf{t}^{\dagger})\rangle = \left\langle \sum_{j=2N_2+1}^{2(N_1+N_2)} \sum_{l=1}^{2N_2} |\widetilde{S}_{jl}|^2 \right\rangle, \qquad (56)$$

where the brackets $\langle \cdots \rangle$ denote an average over the pdf Eq. (46). The conductance variance Var G and the shot noise P can similarly be written as

$$\frac{\mathrm{Var}\, G}{G_0^2} = \langle \{\mathrm{Tr}(\mathbf{t}\mathbf{t}^{\dagger})\}^2\rangle - \langle \mathrm{Tr}(\mathbf{t}\mathbf{t}^{\dagger})\rangle^2 \qquad (57)$$

and

$$\frac{P}{P_0} = \langle \mathrm{Tr}(\mathbf{t}\mathbf{t}^{\dagger}) - \mathrm{Tr}(\mathbf{t}\mathbf{t}^{\dagger}\mathbf{t}\mathbf{t}^{\dagger})\rangle. \qquad (58)$$

The remaining task is to find a relation between the semiclassical parameter ξ and the fictitious time τ. Pandey analyzed hierarchical relations among the eigenvalue correlation functions of random matrices and evaluated the form factor $K(k;\tau)$ (the Fourier transform of the scaled two eigenvalue correlation function). For the random matrices obeying the crossover pdf in Eq. (46), he derived a relation[39]

$$K(k;\tau) = K(k;\infty) + \{K(k;0) - K(k;\infty)\}e^{-4\pi^2\rho^2\tau k} \qquad (59)$$

with $k \downarrow 0$. Here $K(k;0)$ and $K(k;\infty)$ are the form factors of the GOE and GSE random matrices, respectively. They are known to be[37]

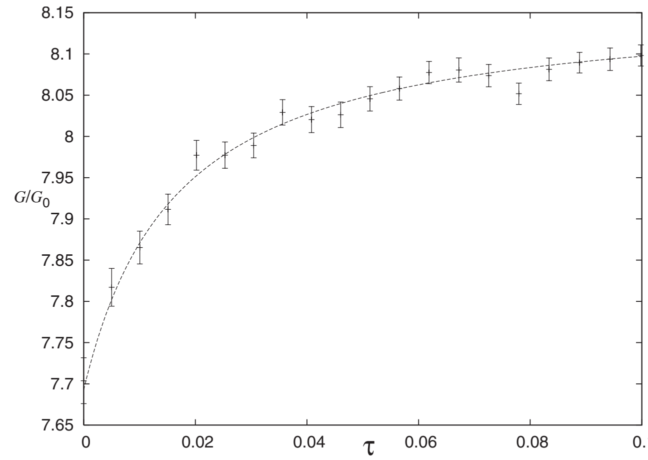

FIG. 5. A comparison of the semiclassical result (curve) and numerically calculated random-matrix results (error bars) for G/G_0.

$$K(k;0) = 2k, \quad K(k;\infty) = \frac{k}{2}, \quad k\downarrow 0 \qquad (60)$$

so that

$$K(k;\tau) = \frac{k}{2}(1 + 3e^{-8M\tau k}), \quad k\downarrow 0. \qquad (61)$$

On the other hand, Nagao and Saito[31] semiclassically analyzed the form factor of a chaotic system with a weak spin-orbit interaction. They obtained a small $k>0$ expansion up to the second order

$$K(k;a) = \frac{k}{2}(1 + 3e^{-akT_H}) + \frac{k^2}{4}\{1 + (3akT_H - 9)e^{-akT_H}\} \qquad (62)$$

with $a = \gamma_{\mathrm{so}}^2 \mathcal{D}$. Comparing Eqs. (61) and (62), we arrive at a relation

$$8M\tau = aT_H. \qquad (63)$$

Therefore the semiclassical parameter $\xi = aT_H/N$ is associated with the random-matrix parameter τ as

$$\xi = \frac{8M}{N}\tau. \qquad (64)$$

In Fig. 5, numerical calculations of G/G_0 at various values of τ are compared with the corresponding semiclassical predictions in Eq. (25) with $\xi = 8M\tau/N$. In the numerical calculations, we set $M=200$, $N_1=20$, and $N_2=5$. The error bars are introduced in order to estimate the statistical errors due to the fact that the averages are calculated over only 300 samples. Note that the semiclassical formulas are truncated and hence are valid only for large N_1 and N_2. Nevertheless we can see a fairly reasonable agreement with the numerical results.

Similar plots for Var G/G_0^2 and P/P_0 are also shown, respectively, in Figs. 6 and 7. The semiclassical curves are drawn by using Eqs. (31) and (35). Since we again find rea-

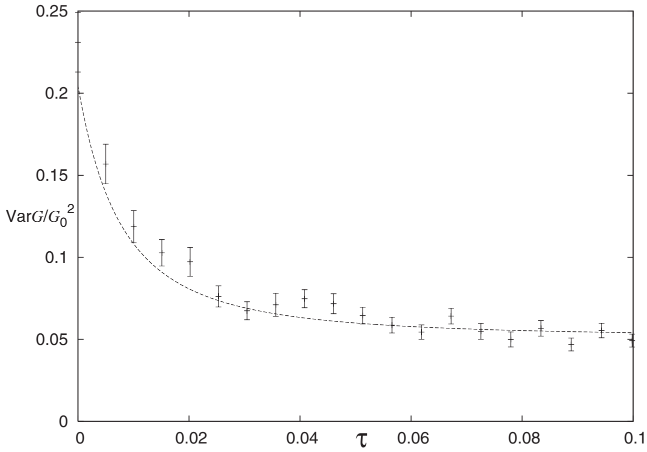

FIG. 6. A comparison of the semiclassical result (curve) and numerically calculated random-matrix results (error bars) for Var G/G_0^2.

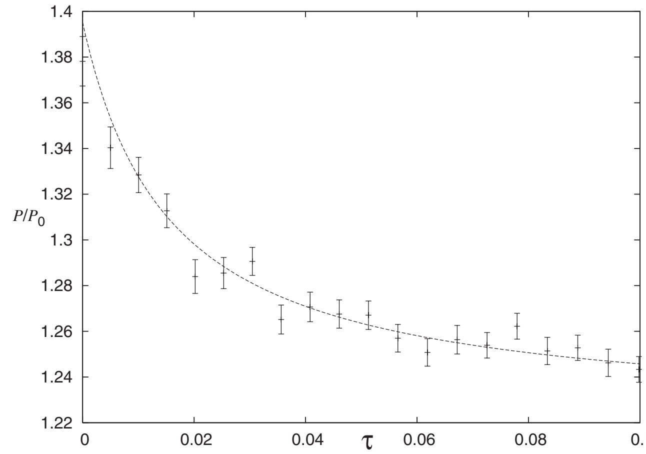

FIG. 7. A comparison of the semiclassical result (curve) and numerically calculated random-matrix results (error bars) for P/P_0.

sonable agreements, it can be conjectured that there is an equivalence between the semiclassical method and random-matrix theory.

In the case of the GOE-GUE crossover, the corresponding random-matrix model can analytically be treated[8] and the results can be compared with the semiclassical formulas.[22] It seems possible to apply similar techniques to the GOE-GSE crossover. For example, the diagrammatic perturbation theory is able to give the leading terms of the transport properties.[14–17] It would be interesting to compare the semiclassical formulas with such analytical results and confirm the equivalence mentioned above.

V. SUMMARY

We studied a chaotic quantum transport of an electron with spin-orbit interaction in a cavity. Our approach is based on the semiclassical theory. The key ingredient of the theory is the universal statistics of the stability amplitudes. The electron diffusion in the position and momentum space is related to the escape rate, and the spin diffuses on the Bloch sphere due to the spin-orbit interaction, where the momentum variable plays the role of a stochastic magnetic field. Consequently the crossover parameter depends on the diffusion constant of the spin. The spin-diffusion terms appear in a non-Abelian way along the classical trajectories. The spins along the trajectories interfere with each other, resulting in the change in the total spin. For instance, Eq. (23) has both singlet and triplet contributions. These kinds of interference effects seem to play a crucial role.

In our calculation of the physical quantities such as the average conductance, only the first several terms of the resulting expansions were worked out. Such truncated results are valid only when the channel numbers are large. In order to obtain the full expansions, a more systematic calculation of the spin-diffusion terms would be necessary. Although our expansions are truncated, they are still useful in the experimental point of view, because the channel numbers can be very large. Moreover the GOE-GSE crossover can be realized when the spin-orbit interaction is controlled by applying an electric field. Therefore we believe that an experimental test of our theory is, in principle possible.

In addition to the transport properties analyzed in this paper, the shot-noise variance is also an important quantity, which can be treated in an RMT framework.[40] It seems possible to apply the semiclassical method to the shot-noise variance. More ambitiously, one might be able to calculate arbitrary order cumulants of the conductance and shot noise in a semiclassical framework, as the RMT approach has already made a progress in that direction.[41]

Our theory is valid in the case that the dwell time of the electron is much larger than the Ehrenfest time. When the Ehrenfest time is relatively large, the resulting corrections should be considered.[42] In addition, the spin-diffusion mechanism might depend on the specific form of the spin-orbit coupling.[15,20] These problems are also interesting in experimentally realizable situations.

ACKNOWLEDGMENTS

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APPENDIX: SPIN-DIFFUSION TERMS

In this appendix, we present the spin-diffusion terms contributing to the average conductance, conductance variance, and shot noise. Here T_j and t_j are the times elapsed on the loop L_j and encounter E_j, respectively, and $a=\gamma_{\mathrm{so}}^2\mathcal{D}$. These spin-diffusion terms were evaluated by using MATHEMATICA.

1. Average conductance

The spin-diffusion terms contributing to the average conductance (corresponding to the diagrams shown in Fig. 2) are listed below.

(i) $$\langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}\Delta_{E_2}\Delta_{L_5}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_4}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_5})^\dagger\}\rangle\rangle$$
$$=\langle\langle \mathrm{Tr}\{\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}(\Delta_{L_3})^{-1}(\Delta_{E_2})^{-1}(\Delta_{L_4})^{-1}\}\rangle\rangle$$
$$=\frac{1}{2}+\frac{3}{2}e^{-a(2t_1+2t_2+2T_2+2T_3)}+\frac{3}{2}e^{-a(2T_2+2T_4)}+\frac{3}{2}e^{-a(2t_1+2t_2+2T_3+2T_4)}-3e^{-a(2t_1+2t_2+2T_2+2T_3+2T_4)}.$$

(ii) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}(\Delta_{E_1})^{-1}\Delta_{L_4}\Delta_{E_2}\Delta_{L_5}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_3})^{-1}(\Delta_{E_2})^{-1}(\Delta_{L_4})^{-1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_5}\}^\dagger]\rangle\rangle$$
$$=\langle\langle \mathrm{Tr}\{\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}(\Delta_{E_1})^{-1}\Delta_{L_4}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_4}\Delta_{E_2}\Delta_{L_3}\}\rangle\rangle=\frac{1}{2}-\frac{3}{2}e^{-a(2t_2+2T_2+2T_3)}-\frac{3}{2}e^{-a(2t_1+2T_2+2T_4)}+\frac{3}{2}e^{-a(2t_1+2t_2+2T_3+2T_4)}$$
$$+3e^{-a(2t_1+2t_2+2T_2+2T_3+2T_4)}.$$

(iii) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}(\Delta_{E_2})^{-1}\Delta_{L_5}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_4}(\Delta_{E_2})^{-1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}(\Delta_{L_3})^{-1}(\Delta_{E_2})^{-1}\Delta_{L_5}\}^\dagger]\rangle\rangle$$
$$=\langle\langle \mathrm{Tr}\{\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}\Delta_{L_3}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}(\Delta_{L_4})^{-1}\}\rangle\rangle=\frac{1}{2}-\frac{3}{2}e^{-a(2t_1+2T_3+2T_4)}+\frac{3}{2}e^{-a(2t_1+2t_2+2T_2+2T_3)}-\frac{3}{2}e^{-a(2t_2+2T_2+2T_4)}$$
$$+3e^{-a(2t_1+2t_2+2T_2+2T_3+2T_4)}.$$

(iv) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}\Delta_{E_2}\Delta_{L_4}(\Delta_{E_2})^{-1}\Delta_{L_5}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_3}\Delta_{E_2}(\Delta_{L_4})^{-1}(\Delta_{E_2})^{-1}\Delta_{L_5}\}^\dagger]\rangle\rangle$$
$$=\langle\langle \mathrm{Tr}\{\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}\Delta_{E_2}\Delta_{L_4}\Delta_{L_4}(\Delta_{E_2})^{-1}(\Delta_{L_3})^{-1}\Delta_{E_1}\Delta_{L_2}\}\rangle\rangle=\frac{1}{2}-\frac{3}{2}e^{-a2T_2}+\frac{9}{2}e^{-a(2T_2+2T_4)}-\frac{3}{2}e^{-a2T_4}.$$

(v) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}(\Delta_{E_2})^{-1}\Delta_{L_4}(\Delta_{E_1})^{-1}\Delta_{L_5}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_4})^{-1}\Delta_{E_2}\Delta_{L_3}(\Delta_{E_2})^{-1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_5}\}^\dagger]\rangle\rangle$$
$$=\langle\langle \mathrm{Tr}\{\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}(\Delta_{E_2})^{-1}\Delta_{L_4}\Delta_{L_2}\Delta_{E_2}(\Delta_{L_3})^{-1}(\Delta_{E_2})^{-1}\Delta_{L_4}\}\rangle\rangle=\frac{1}{2}-\frac{3}{2}e^{-a2T_3}+\frac{3}{2}e^{-a(2T_2+2T_4)}+\frac{3}{2}e^{-a(2T_2+2T_3+2T_4)}.$$

(vi) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_1}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_3}\Delta_{E_1}\Delta_{L_2}\Delta_{E_1}\Delta_{L_4}\}^\dagger]\rangle\rangle=\langle\langle \mathrm{Tr}\{\Delta_{L_2}\Delta_{E_1}\Delta_{L_3}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}(\Delta_{L_3})^{-1}\}\rangle\rangle$$
$$=\frac{1}{2}+\frac{3}{2}e^{-a(2t_1+2T_2)}+\frac{3}{2}e^{-a(2t_1+2T_3)}+\frac{3}{2}e^{-a(2T_2+2T_3)}-3e^{-a(2t_1+2T_2+2T_3)}.$$

(vii) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}(\Delta_{L_3})^{-1}\Delta_{E_1}\Delta_{L_4}\}^\dagger]\rangle\rangle=\langle\langle \mathrm{Tr}\{\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}\Delta_{L_3}\Delta_{E_1}\Delta_{L_2}\}\rangle\rangle$$
$$=\frac{1}{2}-\frac{3}{2}e^{-a2T_2}+\frac{9}{2}e^{-a(2T_2+2T_3)}-\frac{3}{2}e^{-a2T_3}.$$

(viii) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_1}\Delta_{L_3}(\Delta_{E_1})^{-1}\Delta_{L_4}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_3}(\Delta_{E_1})^{-1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_4}\}^\dagger]\rangle\rangle=\langle\langle \mathrm{Tr}\{\Delta_{L_2}\Delta_{E_1}\Delta_{L_3}\Delta_{L_2}\Delta_{E_1}(\Delta_{L_3})^{-1}\}\rangle\rangle$$
$$=\frac{1}{2}+\frac{3}{2}e^{-a(2t_1+2T_2)}+\frac{3}{2}e^{-a(2t_1+2T_2+2T_3)}-\frac{3}{2}e^{-a2T_3}.$$

(ix) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}(\Delta_{E_1})^{-1}\Delta_{L_4}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_3})^{-1}\Delta_{E_1}\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_4}\}^\dagger]\rangle\rangle=\langle\langle \mathrm{Tr}\{\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_3}\}\rangle\rangle$$
$$=\frac{1}{2}-\frac{3}{2}e^{-a2T_2}+\frac{3}{2}e^{-a(2t_1+2T_3)}+\frac{3}{2}e^{-a(2t_1+2T_2+2T_3)}.$$

2. Conductance variance

The spin-diffusion terms contributing to the conductance variance (corresponding to the diagrams shown in Fig. 3) are listed below.

(i) $$\langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{E_1})^{-1}\Delta_{L_3}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_3}\}^\dagger]\mathrm{Tr}\{\Delta_{L_4}(\Delta_{L_4})^\dagger\}\rangle\rangle$$
$$=\langle\langle 2\ \mathrm{Tr}\{(\Delta_{L_2})^2\}\rangle\rangle=-2+6e^{-a2T_2}.$$

$$\text{(ii)}\quad \langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_5}\Delta_{E_2}\Delta_{L_3})^{\dagger}\}\mathrm{Tr}\{\Delta_{L_4}\Delta_{E_1}\Delta_{L_5}\Delta_{E_2}\Delta_{L_6}(\Delta_{L_4}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_6})^{\dagger}\}\rangle\rangle$$

$$=\langle\langle|\mathrm{Tr}\{\Delta_{L_2}(\Delta_{L_5})^{\dagger}\}|^2\rangle\rangle = 1+3e^{-a(2T_2+2T_5)}.$$

$$\text{(iii)}\quad \langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_4})^{\dagger}\}\mathrm{Tr}\{\Delta_{L_5}\Delta_{E_2}\Delta_{L_6}(\Delta_{L_5}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_6})^{\dagger}\}\rangle\rangle$$

$$=\langle\langle|\mathrm{Tr}(\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1})|^2\rangle\rangle = 1+3e^{-a(2t_1+2t_2+2T_2+2T_3)}.$$

$$\text{(iv)}\quad \langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_1}\Delta_{L_3}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_3})^{\dagger}\}\mathrm{Tr}\{\Delta_{L_4}\Delta_{E_1}\Delta_{L_5}(\Delta_{L_4}\Delta_{E_1}\Delta_{L_2}\Delta_{E_1}\Delta_{L_5})^{\dagger}\}\rangle\rangle$$

$$=\langle\langle|\mathrm{Tr}(\Delta_{L_2}\Delta_{E_1})|^2\rangle\rangle = 1+3e^{-a(2t_1+2T_2)}.$$

$$\text{(v)}\quad \langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_5})^{-1}\Delta_{E_2}\Delta_{L_3}\}^{\dagger}]\mathrm{Tr}[\Delta_{L_4}(\Delta_{E_2})^{-1}\Delta_{L_5}(\Delta_{E_1})^{-1}\Delta_{L_6}\{\Delta_{L_4}(\Delta_{E_2})^{-1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_6}\}^{\dagger}]\rangle\rangle$$

$$=\langle\langle\{\mathrm{Tr}(\Delta_{L_2}\Delta_{L_5})\}^2\rangle\rangle = 1+3e^{-a(2T_2+2T_5)}.$$

$$\text{(vi)}\quad \langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_4})^{\dagger}\}\mathrm{Tr}[\Delta_{L_5}(\Delta_{E_2})^{-1}\Delta_{L_6}\{\Delta_{L_5}(\Delta_{E_2})^{-1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}(\Delta_{L_3})^{-1}(\Delta_{E_2})^{-1}\Delta_{L_6}\}^{\dagger}]\rangle\rangle$$

$$=\langle\langle\{\mathrm{Tr}(\Delta_{L_2}\Delta_{E_2}\Delta_{L_3}\Delta_{E_1})\}^2\rangle\rangle = 1+3e^{-a(2t_1+2t_2+2T_2+2T_3)}.$$

$$\text{(vii)}\quad \langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\Delta_{E_1}\Delta_{L_3}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_3})^{\dagger}\}\mathrm{Tr}[\Delta_{L_4}(\Delta_{E_1})^{-1}\Delta_{L_5}\{\Delta_{L_4}(\Delta_{E_1})^{-1}(\Delta_{L_2})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_5}\}^{\dagger}]\rangle\rangle$$

$$=\langle\langle\{\mathrm{Tr}(\Delta_{L_2}\Delta_{E_1})\}^2\rangle\rangle = 1+3e^{-a(2t_1+2T_2)}.$$

$$\text{(viii)}\quad \langle\langle \mathrm{Tr}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{L_1}\Delta_{E_1}\Delta_{L_4})^{\dagger}\}\mathrm{Tr}\{\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}(\Delta_{L_3}\Delta_{E_1}\Delta_{L_2})^{\dagger}\}\rangle\rangle = \langle\langle|\mathrm{Tr}(\Delta_{L_2}\Delta_{L_4}^{\dagger})|^2\rangle\rangle = 1+3e^{-a(2T_2+2T_4)}.$$

3. Shot noise

The spin-diffusion terms contributing to the shot noise (corresponding to the diagrams shown in Fig. 4) are listed below.

$$\text{(i)}\quad \langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{L_3}\Delta_{E_1}\Delta_{L_2})^{\dagger}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}\Delta_{E_2}\Delta_{L_5}(\Delta_{E_2})^{-1}\Delta_{L_6}\{\Delta_{L_1}\Delta_{E_1}\Delta_{L_4}\Delta_{E_2}(\Delta_{L_5})^{-1}(\Delta_{E_2})^{-1}\Delta_{L_6}\}^{\dagger}]\rangle\rangle$$

$$=\langle\langle\mathrm{Tr}\{(\Delta_{L_5})^2\}\rangle\rangle = -1+3e^{-2aT_5}.$$

$$\text{(ii)}\quad \langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}\{\Delta_{L_5}\Delta_{E_2}(\Delta_{L_3})^{-1}\Delta_{E_1}\Delta_{L_2}\}^{\dagger}\Delta_{L_5}\Delta_{E_2}\Delta_{L_4}(\Delta_{E_1})^{-1}\Delta_{L_3}(\Delta_{E_2})^{-1}\Delta_{L_6}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_4})^{-1}(\Delta_{E_2})^{-1}\Delta_{L_6}\}^{\dagger}]\rangle\rangle$$

$$=\langle\langle\mathrm{Tr}\{(\Delta_{E_1}^{\dagger}\Delta_{L_3}\Delta_{L_4})^2\}\rangle\rangle = -1+3e^{-2a(t_1+T_3+T_4)}.$$

$$\text{(iii)}\quad \langle\langle \mathrm{Tr}[\Delta_{L_1}\Delta_{E_1}\Delta_{L_2}(\Delta_{L_3}\Delta_{E_1}\Delta_{L_2})^{\dagger}\Delta_{L_3}\Delta_{E_1}\Delta_{L_4}(\Delta_{E_1})^{-1}\Delta_{L_5}\{\Delta_{L_1}\Delta_{E_1}(\Delta_{L_4})^{-1}(\Delta_{E_1})^{-1}\Delta_{L_5}\}^{\dagger}]\rangle\rangle$$

$$=\langle\langle\mathrm{Tr}\{(\Delta_{L_4})^2\}\rangle\rangle = -1+3e^{-2aT_4}.$$

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